RTD EXAM PREP

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Mathematics 30-2 Diploma Review

Classroom Study Guide Book A

A Few Things about the Diploma Exam

- Bring picture ID to the exam.
- Arrive early for the exam. You'll be less stressed and more prepared as a result. Students who arrive **more than one hour <u>after</u>** a diploma examination has started will **<u>not</u> be allowed to write the examination.**
- If this is a 2nd attempt at writing this diploma exam, you must submit a completed Diploma Examination Registration and Rewrite Fee Form, along with a fee to Alberta Education before the exam date.
- The diploma is designed so that the majority of students can comfortably complete it within 3 hours, but you have **up to 6 hours to complete** it if you need it.
- You **cannot** hand in your exam until **<u>at least one hour</u>** of the examination time has elapsed.
- The diploma exam makes up **30% of your Math 30-2 Course Mark**.

Examination Specifications and Design

32 machine-scored questions (weighted 75%)

- 24 Multiple Choice Questions (1 mark each)
- 8 Numeric Response Questions (1 mark each)

Types of Numerical Response Questions

1. **Correct Solution:** For some numerical-response questions, students are to calculate a numerical answer and record their answer in a separate area of the answer sheet. When the answer to be recorded cannot be a decimal value, students are asked to determine a whole number value:
   e.g., the number of people is _____
   e.g., the degree of this polynomial is _____.
   If the answer can be a decimal value, then students are asked to record their answer to the nearest tenth or nearest hundredth, as specified in the question. **Students should retain all decimals throughout the question, and rounding should occur only in the final answer.**

2. **Correct Order:** usually some sort of matching question where the order of the answer **is important**

Correct-Order Question and Solution

The digits 1 through 6 are arranged in the puzzle below so that the sum of each horizontal row is 7.

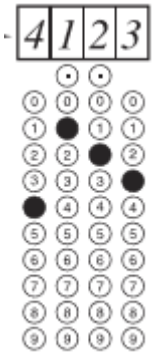
1	***a***
b	2
c	4

In the puzzle above, the value of
a is _____ (Record in the **first** column)
b is _____ (Record in the **second** column)
c is _____ (Record in the **third** column)

(Record your answer in the numerical-response section on the answer sheet.)

Value to be recorded: 653

Record 653 on the answer sheet → 6 5 3



3. Any Order: usually some sort of matching questions where the order of the answer **is not important**

Any-Order Question and Solution

The elements in sets A and B are defined below.

$A = \{2, 3, 4, 5, 7, 8, 9\}$

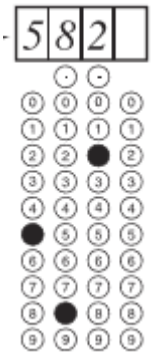
$B = \{1, 2, 5, 6, 8\}$

The three elements that are in Set A and Set B are ___, ___, and ___.

(Record all **three digits** of your answer **in any order** in the numerical-response section on the answer sheet.)

Digits to be recorded: 582 (in any order)

Record 582 on the answer sheet → 5 8 2



2 written-response questions (weighted 25%)

- Questions are equally weighted (7 marks each)
- Will cover more than one specific outcome; need to make connections between concepts
- Each question has four parts.
    - Part 1 (1 mark), Part 2 (2 marks), Part 3 (2 marks) and Part 4 (2 marks)

General Rubric (used to develop specific guide for each WR question)

1-mark Part

Score	General Scoring Guide
NR	No response is provided.
0	In the response, the student does not address the question or provides a solution that is invalid.
0.5	
1	In the response, the student applies appropriate mathematical knowledge to find a complete and correct solution.

- A student response that does not meet the performance level of a benchmark score may receive an augmented score of 0.5 or 1.5.

2-mark Part

Score	General Scoring Guide
NR	No response is provided.
0	In the response, the student does not address the question or provides a solution that is invalid.
0.5	
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.
1.5	
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.

Types of Questions

- 34% will be **conceptual** (compare, contrast, label, define, examples and counterexamples)
- 30% will be **procedural** (recognize, execute and verify appropriate procedures)
- 36% will be **problem solving** (adapt, extend and solve unique and unfamiliar problems)

Topics covered

- 15% - 20% of the exam is **Logic and Reasoning** (Set Theory & Logic)
- 30% - 35% of the exam is **Probability** (Counting Methods & Probability)
- 45% - 55% of the exam is **Functions** (Exponents, Logs, Polynomials, Sine & Rational Functions)

Diploma Exam Tips

General

- Read the question carefully before answering to ensure you know what is being asked.
- Highlight/underline important parts of the question.
- There is plenty of time to complete the exam so **TAKE YOUR TIME**.
- Your first instinct is ***usually*** the right one; don't change it unless you are sure it is wrong.
- If you can't figure out the answer to a question skip it and come back to it later.
- Start answering questions in your strongest unit so that you start off strong and confident.

Multiple Choice

- Try to answer the question first before looking at the options.
- Always eliminate options that don't make sense and narrow your good options down.
- Look for repeated values/answers as they are likely part of the solution.
- Look for options that "stick out" as they are likely **not** the answer.
- Work backwards with the numbers that are given.
- Use your calculator to check/validate the options.
- Don't look for patterns in the answer key (ie: I haven't picked "d" in a while….).
- There might be two answers that you are leaning towards but only one will be correct.
- Your first instinct is usually the correct one; don't overthink it!

Numeric Response

- Numeric response is left-justified; start the leftmost box and work your way right.
- Answers will **never** be negative.
- If it your answer does not fit in the box then it is not the correct answer.
- The decimal gets its own box.
- Pay attention to rounding (tenth means 1 number after the decimal place, hundredth means 2 numbers after the decimal place, whole number means 0 numbers after the decimal place etc.).
- Don't round your answer until the last step.

Written Response

- Do not leave any part of a question blank; attempts at solution may be worth partial marks.
- Show all formulas and steps in your calculations.
- Write legibly and do not use abbreviations of words in your explanation.
- Make sure you use appropriate units and rounding.
- Re-read the question and make sure you have answered all parts of the question.

Directing Words

- Students are required to know the definition of the following directing words.
- These words will be bolded in the written-response questions.
- Pay attention to the directing words.

Algebraically	Using mathematical procedures that involve variables or symbols to represent values
Analyze	Make a mathematical examination of parts to determine the nature, proportion, function, interrelationships, and characteristics of the whole
Classify	Arrange items or concepts in categories according to shared qualities or characteristics
Compare	Examine the character or qualities of two things by providing characteristics of both that point out their mutual *similarities* and *differences*
Conclude	Make a logical statement based on reasoning and/or evidence
Describe	Give a written account of a concept
Design/Plan	Construct a detailed sequence of actions for a specific purpose
Determine	Find a solution, to a specified degree of accuracy, to a problem by showing appropriate formulas, procedures, and/or calculations
Evaluate	Find a numerical value or equivalent for an equation, formula, or function
Explain	Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail
Illustrate	Make clear by giving an example. The form of the example will be specified in the question: e.g., a word description, sketch or diagram
Interpret	Provide a meaning of something; present information in a new form that adds meaning to the original data
Justify	Provide valid reasons, evidence, and/or facts that support a position
Model	Represent a concept of situation in a concrete or symbolic way
Predict	State in advance on the basis of logic
Prove	Establish the truth or validity of a statement by giving factual evidence or logical argument
Sketch	Provide a drawing that represents key features/characteristics of an object or graph
Solve	Give a solution to a problem
Verify	Establish, by substitution or geometric comparison the truth of a statement

Diploma Exam Rules

What you <u>can</u> bring into the exam setting

- Water is allowed but no other food & drinks allowed.
- HB Pencils, pens, erasers.
- Rulers, Protractors & Approved Calculators:
    - Casio: fx 9750 G Plus, fx 9750 GII, fx 9860 GIIs
    - Sharp: EL-9900, EL-9600, EL-9600C
    - Texas Instruments: TI-nspire CX Handheld, TI-83 Plus, TI-83 Plus Silver , TI-84 Plus, TI-84 Plus Silver, TI-84 Plus Pocket SE, or TI-84 Plus C Silver,, TI-nspire (with Touchpad or Clickpad)

What you <u>cannot</u> bring into the exam setting:

- Students may not bring any papers, notes of any kind, or books.
- Students may not bring any headsets, digital audio players, cell phones, pagers, or other electronic devices into the examination room. No hand-held electronic dictionaries and no electronic or paper templates and/or graphic organizers are allowed. Situations where students are in possession of materials not allowed in an examination room will be investigated. This process may lead his/her diploma examination being invalidated.
- Students may not remove any examinations or other examination materials from the examination room. This includes any tear-out pages from the examination booklets and any rough draft materials produced by students.

Diploma results will be available online on "myPass" approximately 3 weeks after writing the exam.

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Success is the sum of small efforts, repeated day in & day out!

Mathematics 30-2 Review Program

Review Concepts

We will review all course concepts you have already learned in class. Consider this a "***refresher course***" to help prepare you for the diploma. Given the amount of material we need to cover with our time constraints, this may feel "***fast***" at times. Notes are laid out in a way to allow students to revisit as needed – you might also consider working ahead between classes if you have the time.

Practice Questions

This is the most important component of your review. We might not have the time to cover ***every*** question in your materials. You are highly encouraged to go back and complete any question in the notes that wasn't done in class. We encourage you to complete all of the practice questions at the end of each section on your own – flipping back to review concepts as needed. Ask your instructor for help on any questions you are struggling with during our work periods. **Answers for each question will be on the top of the next page.**

Class Conduct

When the instructor is talking please pay attention as they are telling you things you need to know for your diploma. Talking in class not only distracts your instructor, but the people around you. During work periods, please be mindful of the learning of others. We take breaks every 1-1.5 hours. If you need to take a break at any time, please feel free to come and go as you please. If you ever find the study environment is distracting, please talk to your instructor and the situation will be dealt with confidentially.

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Remember, there's a fine line between a numerator and a denominator!

Mathematics 30-2 Diploma Review

Rational Functions & Equations Topics:

Curriculum Checklist

Outcomes: You should be able to:
- determine equivalent forms of rational expressions
- perform operations (+/-/×/÷) on rational expressions
- solve problems involving rational equations

Acceptable level:
- simplify a rational expression
- determine if two expressions are equivalent
- identify errors when simplifying or +/-/×/÷ rational expressions
- state NPV for rational expression, explain why NPV exists and demonstrate by substitution a value is a NPV
- +/- two rational expressions with same/different monomial denominators
- +/- two rational expressions with equivalent binomial denominators
- ×/÷ two rational expressions
- state NPV for +/-/×/÷
- identify error in operations with rational expressions
- solve rational equations that simplify to a linear equation and identify extraneous solutions
- solve word problems (given the equation) that simplify to a linear equation and identify extraneous solutions
- identify errors in solving rational equations
- explain extraneous solutions in a word problem

Standard of excellence:
- determine an equivalent rational expression, given a rational expression and NPVs
- identify **and correct** errors when simplifying or +/-/×/÷ rational expressions
- +/- two rational expressions that have **different** binomial denominators
- explain why dividing rational expressions may produce additional NPVs for the quotient
- solve rational equations that simplify to a **quadratic** equation and identify extraneous solutions
- solve word problems (**not given the equation**) that simplify to a quadratic equation and identify extraneous solutions
- identify **and correct** errors in solving rational equations

Notes:
- rational expressions **start** with monomial and binomial numerators/denominators but could end up with quadratic numerators/denominators **after** performing operations
- will **only** do one operation on rational expressions
- quadratics can be solved using a variety of methods (factoring or quadratic formula)
- watch out for extraneous solutions to equations and word problems
- state NPV **before** simplifying
- (x – a) and (a – x) have equivalent binomial factors

Topic 1: Review of Factoring

- Factoring means to write a polynomial as the product of simpler polynomials.
- **Example**: $12x^3 - 45x^2 = 3x^2(4x - 15)$

Factoring is a key skill for rational functions and equations. You will need to know how to factor by greatest common factor and difference of squares.

1.1 Greatest Common Factor

- **Always look for this first!**
- Find the GCF of both terms and place it outside the brackets.
- Divide each term inside the brackets by the GCF and write the result inside the brackets.
- Pay attention to the signs!

Calculator Tip

The graphing calculator can help you find the **GCF** of two numbers.

- Math → Num →GCD (9)

Enter the two **positive** numbers separated by a comma (above the number 7).

RTD TIP:

- the **GCF** of two variables is the **common** variable with the **smallest** exponent.
- ***example***: $gcf(x^3, x^2) = x^2$

Factoring Question 1: Factor the following completely.

a. $2a^4 + 8a$	**b.** $5x^3 - 10$
c. $-32y^2 - 24y$	**d.** $10d^2 - 15d^3$
e. $10d^7 + 2d$	**f.** $7w^5 - 35w^2$

Answer to #1: ***a.*** $2a(a^3+4)$ ***b.*** $5(x^3-2)$ ***c.*** $-8y(4y+3)$ ***d.*** $5d^2(2-3d)$ ***e.*** $2d(5d^6+1)$ ***f.*** $7w^2(w^3-5)$

1.2 Difference of Squares $a^2 - b^2$

- Difference of squares are two terms subtracted; no middle term.
- Place the square root of the first number as the first term in each bracket, the square root of the second number as the second term in each bracket and separate them by + and – in any order.

$$\sqrt{a^2} = a \qquad a^2 - b^2 \qquad \sqrt{b^2} = b$$

first term *second term*

$$(a+b)(a-b)$$

RTD TIP: Variables are perfect squares if the exponents are even.

- $\sqrt{variable^{exponent}} = variable^{exponent \div 2}$
- $\sqrt{x^6} = x^3$ because $6 \div 2 = 3$

Calculator Tip

The graphing calculator can help you make a list of **perfect squares**.

- Y1 = x^2
- table (2nd graph)
- all the values in Y1 are perfect squares

Square Roots ← → **Perfect Squares**

X	Y1
0	0
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100

RTD TIP: Numbers are perfect squares if their square roots are rational numbers.

- 25 **IS** a perfect square since $\sqrt{25} = 5$
- 26.01 **IS** a perfect square since $\sqrt{26.01} = 5.1$
- 27 is **NOT** a perfect square since $\sqrt{27} = 5.196152423\ldots$

Factoring Question 2: Factor the following completely. Always look for GCF first!

a. $x^2 - 36$	**b.** $4n^2 - 49$
c. $36k^2 - 4$	**d.** $196n^2 - 144$
e. $18m^3 - 32m$	**f.** $20a^2 - 45$

Answer to #2: $\boldsymbol{a}.\ (x+6)(x-6)$ $\boldsymbol{b}.\ (2n+7)(2n-7)$ $\boldsymbol{c}.\ 4(3k+1)(3k-1)$ $\boldsymbol{d}.\ 4(7n+6)(7n-6)$ $\boldsymbol{e}.\ 2m(3m+4)(3m-4)$ $\boldsymbol{f}.\ 5(2a+3)(2a-3)$

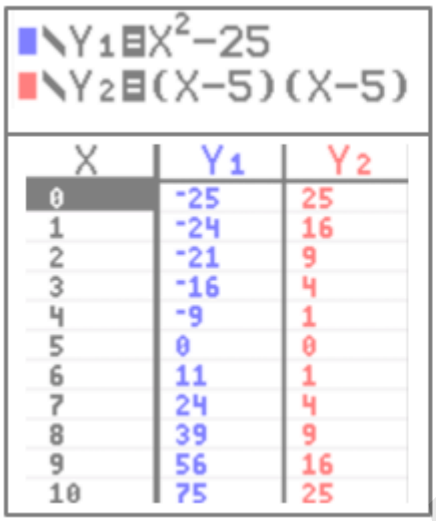
Calculator Tip

The graphing calculator can help you **check** if you factored correctly.

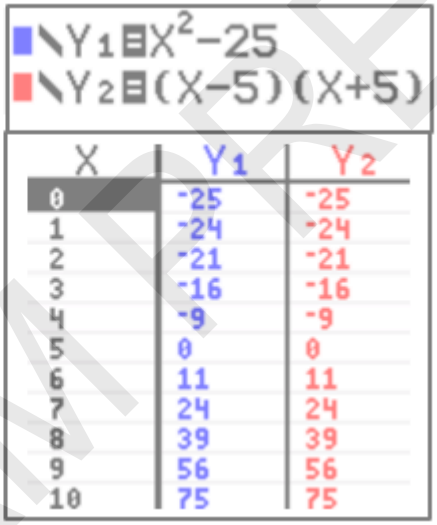
- Enter the original expression into Y1.
- Enter the factored expression into Y2.
- Press 2nd **GRAPH** and look in the table.
  - If Y1 = Y2, the expression is factored correctly.
  - If Y1 ≠ Y2, the expression is factored incorrectly.

RTD NOTE:
- this only works with **one** variable

Example: Does $x^2 - 25 = (x-5)(x-5)$ or does $x^2 - 25 = (x-5)(x+5)$?



X	Y1	Y2
0	-25	25
1	-24	16
2	-21	9
3	-16	4
4	-9	1
5	0	0
6	11	1
7	24	4
8	39	9
9	56	16
10	75	25



X	Y1	Y2
0	-25	-25
1	-24	-24
2	-21	-21
3	-16	-16
4	-9	-9
5	0	0
6	11	11
7	24	24
8	39	39
9	56	56
10	75	75

1.3 Leading Coefficient

- The number in front of the variable term with the highest exponent is the leading coefficient.
- When factoring you want to leave the resulting polynomial with a **positive** leading coefficient.

Factoring Question 3: Factor each binomial so that the leading co-efficient is positive.

a. *3 – y*	***b.*** *$-20x^3 - 15x$*	***c.*** *$18y - 27y^3$*

Answer to #3: $a.-(y-3)$ $b.-5x(4x^2+3)$ $c.-9y(3y^2-2)$

Topic 2: Rational Expressions

- Polynomials divided by polynomials (numbers and variables combined by +, – and ×).
- **Examples**: $\frac{3x-9}{x^2-3}$, $\frac{x}{2x^2}$, $2x-1$

2.1 Non-Permissible Values

- It is "**OK**" for the numerator to be zero.
- A zero in the denominator is a big "**NO**"!
- NPVs are the values of the variable that make the denominator equal to zero.
- To algebraically find the NPVs, set the denominator equal to zero, factor the denominator, set each factor to zero and solve for the variable.
- The NPVs of a rational function are the x-intercepts of the denominator.
- When dealing with rational expressions and equations state the NPV as **restrictions**.

O **N**
K **O**

Expressions Question 1: State the non-permissible values and restrictions of the following.

$$\frac{t+5}{3t^2-t}$$

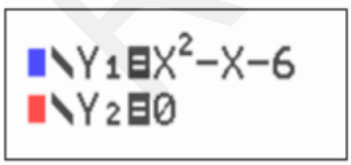
RTD TIP: The highest exponent of the ***variable*** tells you the **maximum** number of NPVs.

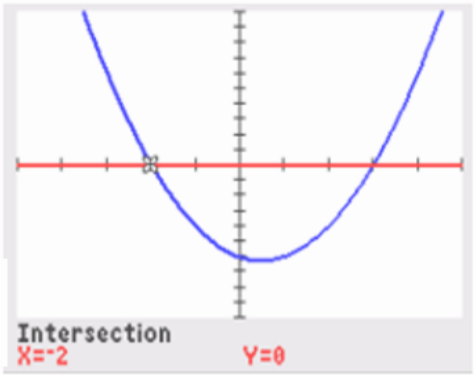
Calculator Tip

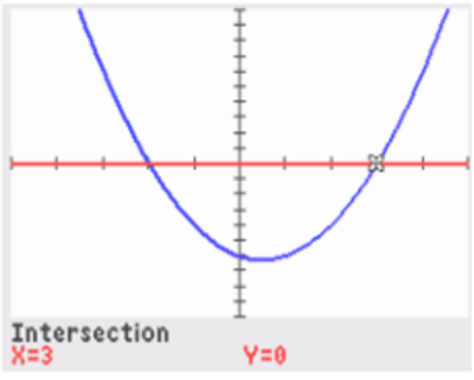
Since the **NPV** of a rational are the x-intercepts of the denominator, the graphing calculator can help you find **NPVs:**

- Enter the entire denominator into Y1 and zero into Y2 (the x-axis is $y=0$).
- Press **GRAPH**.
- To calculate the first x-intercept, press **2nd TRACE**, **5** (INTERSECT), toggle over to the first x-intercept and press **ENTER**, **ENTER**, **ENTER**.
- Repeat the process for all of the x-intercepts.

Example: $\frac{x+4}{x^2-x-6}$







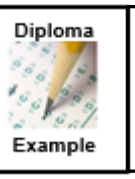
NPV: $-2, 3$
Restrictions: $x \neq -2, 3$

Answer to #1: NPV: $0, \frac{1}{3}$ $Restrictions$: $t \neq 0, \frac{1}{3}$

Expressions Question 2: State the NPV as restrictions for each expression.

a. $\frac{16x^3}{10x}$	**b.** $\frac{4a^2}{a^2+a}$	**c.** $\frac{b-2}{2b^2-8}$

RTD TIP: Monomial denominators **always** have restrictions variable ≠ 0.



The non-permissible value of *x* in the expression $\frac{2x+1}{3x-9}$ is _____.

2.2 Simplifying Rational Expressions

- **F**actor numerator and denominator completely (**positive leading coefficient**).
- State non-permissible values as **R**estrictions.
- **S**implify by removing common **factors** (not common terms).
- Leave the answer in factored form.

F
R
S

Expressions Question 3: Simplify the following. State the NPV as restrictions.

a. $\frac{2x^2-2}{x+1}$	**b.** $\frac{6m^2-8m}{3m^3-4m^2}$

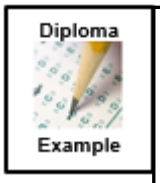
Answer to #2*a*. $x \neq 0$ ***b***. $a \neq -1,0$ ***c***. $b \neq \pm 2$ **Answer to DE**: 3 **Answer to #3*a***. $2(x-1), x \neq -1$ ***b***. $\frac{2}{m}, m \neq 0, \frac{4}{3}$

RTD NOTE:

- same terms, same sign simplifies to +1
  - $\frac{a+b}{b+a} = 1$
- same terms, opposite sign simplifies to -1
- put the -1 in the numerator
  - $\frac{a+b}{b-a} = -1$

Expressions Question 4: Simplify the following. State the NPV as restrictions.

a. $\frac{1-x}{x^2-1}$	**b.** $\frac{2c-8}{16-4c}$



Susan and Dan both simplified the expression $\frac{x}{x^2+x}$. Their work is shown below.

Susan	**Dan**
$\frac{x}{x^2+x}$	$\frac{x}{x^2+x}$
$\frac{x}{x(x+1)}$	$\frac{x}{x^2+x}$
$\frac{1}{x+1}$	$\frac{1}{x+1}$

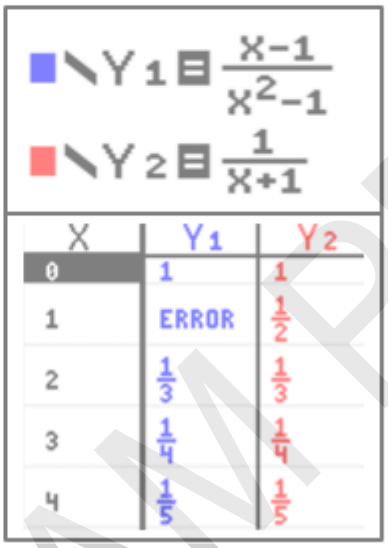
Susan stated that the non-permissible values are -1 and 0. Dan stated that since you can remove a factor of *x* that the non-permissible value is -1. Which student is correct?

Answer to #4*a*. $\frac{-1}{x+1}, x \neq \pm 1$ ***b***. $-\frac{1}{2}, c \neq 4$ **Answer to DE:** Susan is correct; state restrictions **before** simplifying.

Calculator Tip

If you ever want to check if two expressions are equivalent, you can use the graphing calculator to help you check.

- Enter one expression into Y1 and the other expression into Y2.
- Press 2nd GRAPH and look in the table.
  - If **Y1 = Y2**, the expressions **are equivalent** for all *permissible* values.
  - If **Y1 ≠ Y2** the expressions **are not equivalent.**



RTD NOTE:

ALPHA Y= gives you the fraction template $\frac{\square}{\square}$.

Diploma Example

Which of the following expressions is equivalent to $\frac{5}{3x+5}, x \neq \frac{-5}{3}, 0, \frac{5}{3}$?

A. $\frac{5x(3x-5)}{x(3x-5)(3x+5)}$

B. $\frac{5x(3x+5)}{x(3x-5)(3x+5)}$

C. $\frac{5(3x-5)}{x(3x-5)(3x+5)}$

D. $\frac{5}{x(3x-5)(3x+5)}$

RTD NOTE: It would be easier to work backwards from the options.

Answer to DE: A.

Diploma Example

The simplified form of the expression $\frac{3x^2 - 3x}{x^2 - 1}$, and the restriction on x, are, respectively,

A. $3 + 3x$ and $x \neq 1$

B. $3 + 3x$ and $x \neq -1, 1$

C. $\frac{3x}{x+1}$ and $x \neq -1$

D. $\frac{3x}{x+1}$ and $x \neq -1, 1$

RTD NOTE: Questions like these are "gifts". You could simply use your graphing calculator to find the answer.

Diploma Example

Use the following information to answer the next question.

Ken made an error in the simplification of the rational expression $\frac{2x + 10}{2x^2 - 50}$, $x \neq -5, 5$. His simplification of the expression is shown below.

Step 1 $\frac{{}^{1}\cancel{2}(x+5)}{{}^{1}\cancel{2}(x^2 - 25)}$

Step 2 $\frac{1(x+5)}{1(x+5)(x-5)}$

Step 3 $\frac{1\cancel{(x+5)}}{1\cancel{(x+5)}(x-5)}$

Step 4 $(x-5), x \neq -5, 5$

The step in which Ken made his error is

A. Step 1
B. Step 2
C. Step 3
D. Step 4

Diploma Example

An expression with a non-permissible value of $x = 1$ has been simplified to x. Determine an equivalent rational expression for the simplified expression.

Answer to DE: D **Answer to DE:** D **Answer to DE:** Answers vary. One possible answer: $\frac{x(x+1)}{(x-1)}$

Diploma Example

Use the following information to answer numerical-response question

An expression equivalent to $\frac{3x-8}{x+3}$, $x \neq -3, 0$, is written in the form $\frac{Ax^B - 16x}{Cx^2 + Dx}$, where A, B, C, and D represent single-digit whole numbers.

Numerical Response

The value of

A is _____ (Record in the **first** column)

B is _____ (Record in the **second** column)

C is _____ (Record in the **third** column)

D is _____ (Record in the **fourth** column)

2.3 Multiplying Rational Expressions

- **F**actor numerator and denominator completely (**positive leading coefficient**).
- State non-permissible values as **R**estrictions.

$\frac{a}{b} \times \frac{c}{d}$ **b, d ≠ 0**

- **M**ultiply the numerators together and denominators together.
- **S**implify by removing common **factors** (not common terms).
- Leave the answer in factored form.

F
R
M
S

Expressions Question 5: Multiply the following. State the NPV as restrictions.

a. $\frac{10x}{4y} \times \frac{2y}{5}$	**b.** $\frac{8x^2}{25} \times \frac{50}{2x^3}$

RTD TIP: After you multiply, simplify by dividing numbers with numbers, x with x, y with y, etc.

Answer to DE: 6226 **Answer to #5*a*.** $x, y \neq 0$ ***b*.** $\frac{8}{x}, x \neq 0$

Diploma

Example

Use the following information to answer the next question.

The simplified product of $\frac{2n^4p}{3m} \cdot \frac{6m^6}{3n^2p^2}, m \neq 0, n \neq 0, p \neq 0,$ can be represented by

$$\frac{\boxed{A}n^{\boxed{B}}m^{\boxed{C}}}{3p}$$

where ***A***, ***B***, and ***C*** represent single-digit numbers.

Numerical Response

In the simplified product $\frac{\boxed{A}n^{\boxed{B}}m^{\boxed{C}}}{3p}$, the value of

A is _____ (Record in the **first** column)

B is _____ (Record in the **second** column)

C is _____ (Record in the **third** column)

F R M S

Expressions Question 6: Multiply and state the NPV as restrictions.

a. $\frac{6y-30}{y-1} \times \frac{5y-5}{3y^2-15y}$

b. $\frac{6x^2-3x}{7x^3-42x^2} \times \frac{84-14x}{8x^3}$

Answer to DE: 425 **Answer to #6*a***. $\frac{10}{y},\ y \neq 0, 1, 5$ ***b***. $\frac{-3(2x-1)}{4x^4},\ x \neq 0, 6$

Diploma

Example

Use the following information to answer numerical-response question

A rational expression in the form $\frac{A}{B} \cdot \frac{3m^3}{6n^2}$ can be simplified to $\frac{m}{n}$, where $m \neq 0,\ n \neq 0$. Celine knows that the original rational expression can be formed by selecting expressions for A and B from the tables below.

Possible Expression for A	Code
n^2	1
n^3	2
$2n^2$	3
$2n^3$	4

Possible Expression for B	Code
nm	5
nm^2	6
n^2m^2	7

Numerical Response

Record the code numbers that identify the expressions Celine can select for ***A*** and ***B*** to form the original rational expression. (There is more than one correct answer.)

Code Number: _____ _____
Expression: ***A*** ***B***

2.4 Dividing Rational Expressions

1. Factor the numerator and denominator completely (keep the leading coefficient positive).
- State the non-permissible values as restrictions.

$\frac{a}{b} \div \frac{c}{d}$ **b, c, d ≠ 0**

RTD TIP:

Keep **C**hewbacca **F**ly
Keep the 1st fraction the same
Change ÷ to ×
Flip the 2nd fraction

- Multiply fraction 1 by the reciprocal of fraction 2; watch for **NEW restriction**!
- Simplify and leave your answer in factored form.

Answer to DE: 36 *or* 47

Diploma Example

The number of distinct non-permissible values in the expression $\frac{(x-1)(x+7)}{(x+7)(x-9)} \div \frac{x}{(x-9)(x+8)}$ is

A. 5
B. 4
C. 3
D. 2

KCF

Expressions Question 7: Divide the following. State the NPV as restrictions.

a. $\frac{6x^2}{15} \div \frac{18x}{15}$

b. $\frac{-6y^2}{18} \div \frac{3y^3}{14}$

RTD TIP: No variable, no restriction.

Expressions Question 8: Divide the following. State the NPV as restrictions.

a. $\frac{x-2}{3x^2-9x} \div \frac{5}{6x-18}$

b. $\frac{x+5}{x^2-25} \div \frac{25}{5-x}$

Answer to DE: B **Answer to #7*a*.** $\frac{x}{3}, x \neq 0$ ***b*.** $\frac{-14}{9y}, y \neq 0$ **Answer to #8*a*.** $\frac{2(x-2)}{5x}, x \neq 0,3$ ***b*.** $\frac{-1}{25}, x \neq \pm 5$

Diploma Example

The non-permissible values of x for the expression $\frac{x+3}{x-4} \div \frac{x}{x-3}$ are

A. –3, 0, 3, and 4

B. 0, 3, and 4

C. 0 and 4

D. 3 and 4

Diploma Example

Use the following information to answer the next question.

Jenny was asked to simplify

$$\frac{x+1}{x^2-1} \div \frac{x+1}{x-1}, x \neq -1, 1$$

Which of the following is a correct simplification of this expression?

A. $\frac{{}^{1}\cancel{x+1}}{{}^{1}\cancel{(x-1)}(x+1)} \cdot \frac{{}^{1}\cancel{x-1}}{{}^{1}\cancel{x+1}} = \frac{1}{x+1}$

B. $\frac{{}^{1}\cancel{x+1}}{(x-1)\cancel{(x+1)}^{1}} \cdot \frac{x+1}{x-1} = \frac{x+1}{(x-1)^2}$

C. $\frac{{}^{1}\cancel{x+1}}{{}^{1}\cancel{(x-1)}(x-1)} \cdot \frac{{}^{1}\cancel{x-1}}{{}^{1}\cancel{x+1}} = \frac{1}{x-1}$

D. $\frac{x+1}{(x-1)(x-1)} \cdot \frac{x+1}{x-1} = \frac{(x+1)^2}{(x-1)^3}$

Answer to DE: B **Answer to DE:** A

2.5 Adding and Subtracting Rational Expressions

Lowest Common Multiple

- To add and subtract fractions you need a common denominator.
- To find a common denominator you need to find the LCM of both denominators.
- **Example**: $LCM(12x^3, 45x^2) = 180x^3$

Calculator Tip

The graphing calculator can help you find the **LCM** of two numbers.

- Math → Num →LCM (8)

Enter the two **positive** numbers separated by a comma (above the number 7).

RTD TIP:

- the **LCM** of two variables is the **common** variable with the **largest** exponent.
- ***example***: $lcm(x^3, x^2) = x^3$

Expressions Question 9: Find the LCM of the following denominators.

a. $\frac{3w}{12} - \frac{12}{5w}$	**b.** $\frac{1}{12n^2} + \frac{n+1}{9n}$
c. $\frac{4}{k+2} + \frac{8k}{k-8}$	**d.** $\frac{z}{2z^2-2} - \frac{1}{3z+3}$

RTD TIP: LCM of binomial factors is one of **each** factor.

Adding and Subtracting Rational Expressions

- Factor the denominators only, find the LCM and state all non-permissible values.
- Create **equivalent fractions** with the LCM as the new denominator for each fraction.
- Leave the LCM in factored form and add or subtract the numerators by combining like terms.
- Factor the numerator, if possible, and simplify.

RTD TIP: When subtracting fractions, subtract the **ENTIRE** numerator of the second fraction **from** the numerator of the first fraction.

Answer to #9: $\boldsymbol{a}.\ 60w \quad \boldsymbol{b}.36n^2 \quad \boldsymbol{c}.\ (k+2)(k-8) \quad \boldsymbol{d}.6(z+1)(z-1)$

Expressions Question 10: Add/subtract the following. State any restrictions.

a. $\frac{3}{5x}+\frac{7x}{4}$	**b.** $\frac{5x+1}{x-1}-\frac{3x+1}{x-1}$
c. $\frac{24}{x^2-16}+\frac{3}{x+4}$	**d.** $\frac{2}{x-2}-\frac{3}{x}$

***Expressions Question* 11:** Two rational functions have a sum of $\frac{11x+7}{x^2+x}$. One rational function is $\frac{7}{x}$ and the other rational function has the form $\frac{A}{x+B}$. What are the values of A and B?

Answer to #10*a*. $\frac{35x^2+12}{20x}, x \neq 0$ ***b***. $\frac{2x}{x-1}, x \neq 1$ ***c***. $\frac{3}{x-4}, x \neq \pm 4$ ***d***. $\frac{-x+6}{x(x-2)}, x \neq 0,2$ **Answer to #11:** $A = 4 \; and \; B = 1$

Diploma Example

When $\frac{x}{3x+12} + \frac{x-1}{6x+24}$, $x \neq -4$, is simplified, the numerator is $3x - 1$ and the denominator is

A. $6(x+4)$
B. $6(x+4)^2$
C. $18(x+4)$
D. $18(x+4)^2$

RTD TIP:

- You could use your calculator for this question.
  - Y1 = the original expression
  - Y2 = $\frac{3x-1}{each\ option\ (a,b,c,d)}$
- Check each option in the table until Y1 = Y2.

Diploma Example

Use the following information to answer the next question.

The simplified sum of $\frac{x+4}{3x} + \frac{1}{2}, x \neq 0$, can be written in the form

$$\frac{\boxed{A}x + \boxed{B}}{\boxed{C}x}$$

where A, B, and C represent single-digit numbers.

In the simplified sum $\frac{\boxed{A}x + \boxed{B}}{\boxed{C}x}$, the value of

A is _____ (Record in the **first** column)
B is _____ (Record in the **second** column)
C is _____ (Record in the **third** column)

Diploma Example

Which of the following expressions is equivalent to $\frac{1}{x+2} - \frac{4}{x}$, where $x \neq -2, 0$?

A. $\frac{5x+8}{x(x+2)}$

B. $\frac{5x+2}{x(x+2)}$

C. $\frac{-3x+2}{x(x+2)}$

D. $\frac{-3x-8}{x(x+2)}$

Answer to DE. A **Answer to DE.** 586 **Answer to DE.** D

Topic 3: Review of Solving Equations

- Solving a rational equation will result in a linear or quadratic equation that needs to be solved.

RTD TIP: If the equation is not linear or quadratic look for a **GCF** that can be factored.

3.1 Linear Equations

- Equations in which the variable has no exponent or an "invisible" exponent of one.
- Use inverse operations to isolate the variable.
  - Variable on one side of the equation and number on the other.

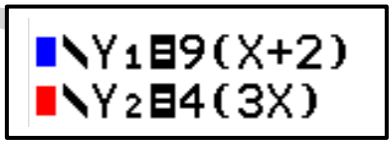
***Review Question* 1**: Solve the following equations.

a. $9(x+2)=4(3x)$	**b.** $(x+7)+3(2x)=-2(x+7)$

RTD TIP: ALWAYS check your answer for any equation you solve! Either substitute the value into the original equation to see if LS = RS or use your calculator to do the check.

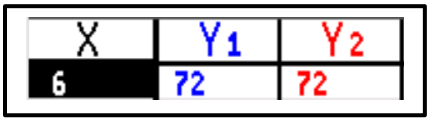
Calculator Tip

Enter the left side (LS) of the original equation into Y1 and the right side (RS) of the original equation into Y2.



Use Table set (2nd WINDOW) to start the table at the answer you are checking.

In table (2nd GRAPH) if Y1 = Y2 for that value, your answer is correct. If Y1 ≠ Y2 your answer is incorrect



Answer to #1a. $x = 6$ **1b.** $x = -\frac{7}{3}$

3.2 Quadratic Equations

- Quadratic equations are equations in which the variable has an exponent of two.
  - Use inverse operations to get the **entire** equation on one side and **zero** on the other.
  - Use the quadratic formula or factoring to solve for x.
    - $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is on the formula sheet

RTD TIP: $ax^2 + bx + c = 0$
- if $a = 1$, factor
- if $a \neq 1$, quadratic formula

***Review Question* 2**: Solve the following equations.

a. $2x + (x - 3) = x(x - 1)$

b. $(x - 3)(x + 1) = 3x(x + 5)$

Topic 4: Rational Equations

- Equations that have a variable in the numerator and/or denominator.
- **Example:** $\frac{18}{x^2 - 3x} = \frac{6}{x - 3} - \frac{5}{x}$
- Always list NPV and check them against your final solution(s).

Extraneous Roots

- Solutions to the linear or quadratic equation that are **NOT** solutions to the **original** equation.
- Always check solutions against the NPV and in the original equation.
- Pay attention to extraneous roots when solving word problems.
  - Time, speed, distances, etc.… can't be negative.
  - Number of objects and people can't be decimals.

Answer to #2*a*. $x = 1, 3$ ***b***. $x = \frac{-17 \pm \sqrt{265}}{4}$

4.1 Fraction = Fraction: Cross products

- Multiply the numerator of Fraction 1 by the denominator of Fraction 2.
- Multiply the denominator of Fraction 1 by the numerator of Fraction 2.
- Set the products equal and solve.

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a}{b} \times \frac{c}{d}$$

$$a \times d = b \times c$$

***Equations Question* 1**: Solve the following equations.

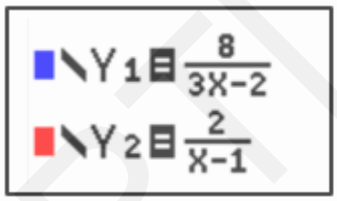
a. $\frac{8}{3x-2} = \frac{2}{x-1}$

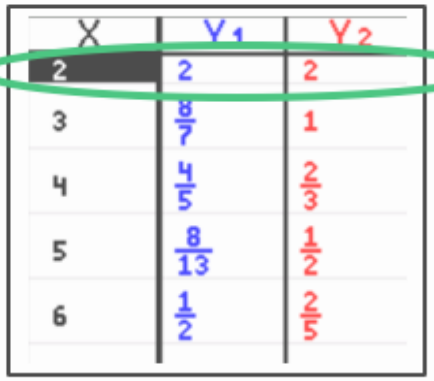
b. $\frac{x+4}{6x} = \frac{4}{3x}$

Calculator Tip

The graphing calculator can help you check your answer.

- Enter the left side of the equation in **Y1**.
- Enter the right side of the equation in **Y2**.
- Start the table (**2nd WINDOW**) at the solution you want to check.
- Go into the table (**2nd GRAPH**).
  - If Y1 = Y2 for the x-value you are checking, then you are correct.





Answer to #1a. $x = 2$ ***b***. $x = 4$

4.2 Fraction ≠ Fraction: Eliminate the denominators

- Factor the denominators, find the LCM and state NPV as restrictions.
- Multiply **each term** of the equation by the LCM to eliminate **all the fractions.**
- Solve the resulting linear or quadratic equation and check the solution(s).

Equations Question 2: Solve the following equations.

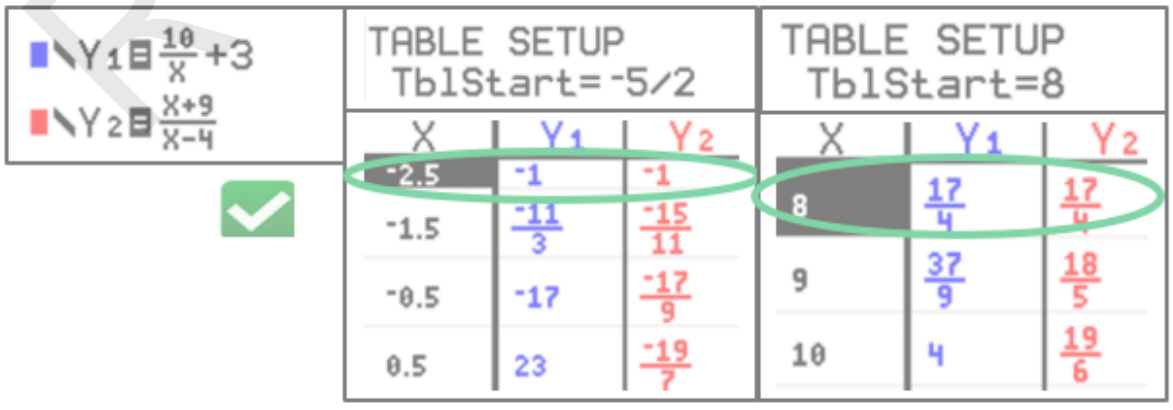
a. $\frac{10}{x} + 3 = \frac{x+9}{x-4}$

b. $\frac{18}{x^2 - 3x} = \frac{6}{x-3} - \frac{5}{x}$

Calculator Tip

The graphing calculator can help you check your answer.

- Enter the left side of the equation in **Y1**.
- Enter the right side of the equation in **Y2**.
- Start the table (**2nd WINDOW**) at the solution you want to check.
- Go into the table (**2nd GRAPH**).
  - If Y1 = Y2 for the x-value you are checking, then you are correct.



Answer to #2*a*. $x = -\frac{5}{2}, 8$ ***b***. *no solution*

Diploma Example

The solution for x in the equation $\frac{-2}{3} - \frac{4}{x} = 6, x \neq 0,$ is

A. –15

B. –9

C. $\frac{-3}{5}$

D. $\frac{3}{5}$

RTD TIP:

- You could check each of the options in the original equation to see which one is correct!

Diploma Example

Use the following information to answer question

While correctly solving the rational equation $\frac{3x+6}{5} + \frac{5x}{x+2} = 2x$ algebraically, a student wrote an equivalent quadratic equation of the form $ax^2 + bx + c = 0$.

The equivalent quadratic equation could have been

A. $2x^2 + 6x - 6 = 0$

B. $3x^2 + 35x + 12 = 0$

C. $7x^2 - 17x - 12 = 0$

D. $10x^2 + 12x - 6 = 0$

Answer to DE: C **Answer to DE:** C

4.3 Word Problems

- Identify and define the unknown.
- Use a table (Distance Speed Time or Job Share questions) to organize the information given in the question.
- State the equation and solve it algebraically (written response) or using your graphing calculator (multiple choice or numeric response).
- Rule out inadmissible solutions:
    - Can't have negative values for time, distance, etc.
    - Can't have decimal values for the number of items.
    - Can't have solutions that match the NPV.

Equations Question 3: Matteo drove 404 km from Edmonton to Banff in the same length of time Amber took to drive 364 km from Edmonton to Jasper. Matteo drove 10 km/h faster than Amber.

	Distance (km)	Speed (km/h)	Time (h)
Matteo			
Amber			

RTD TIP:
FILL, FILL, CALCUALTE

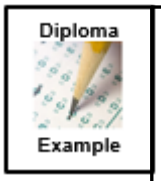
- Fill in first 2 columns.
- Calculate time using the formula. $t = \frac{d}{s}$ and the values you filled in.
- Read the question to find the equation for time.

Determine Matteo's speed, in kilometers per hour.

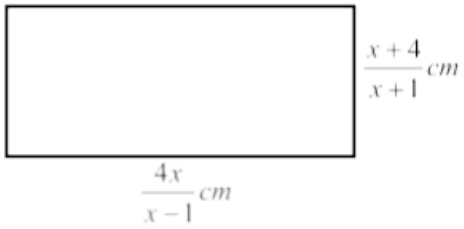
Answer to #3: Matteo drove 101 km/h.

Equations Question 4: When working alone, it takes Jill 3h to paint a room and Jack 2h to paint a room that is the same size. How long would it take them if they worked together?

	Time alone to complete job	Fraction of Job completed in 1h	Fraction of job completed in "x" hours
Jill		$\frac{1}{time\ alone} = \_\_\_\_\_$	
Jack		$\frac{1}{time\ alone} = \_\_\_\_\_$	
Both		$\frac{1}{time\ both} = \_\_\_\_\_$	**Always 1**



The dimensions of a rectangle are represented by rational expressions, where $x > 1$, as shown in the diagram below.



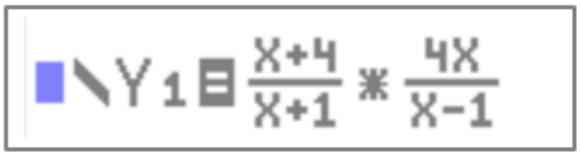
If the area of the rectangle is 16cm^2, determine the dimensions of the rectangle to the nearest centimeter.

Answer to #4: $\frac{6}{5}$ hours for both. **Answer to DE:** 2cm by 8cm

Calculator Tip

The graphing calculator could have helped you solve the last question

- Enter (length)(width) in **Y1**
- Start the table (**2nd WINDOW**) at 1 since the question stated $x > 1$
- Look in the table (**2nd GRAPH**) and find the x-value that gives an area of 16



X	Y1
1	ERROR
2	16
3	21/2
4	128/15
5	15/2

$\frac{(2)+4}{(2)+1} = 2cm$

$\frac{4(2)}{(2)-1} = 8cm$

Diploma Example

Use the following information to answer the next question.

Over a distance of 800 km, the average speed of a small airplane is 6 times faster than the average speed of a train. This information is shown in the table below.

	Distance (km)	Speed (km/h)	Time (h)
Train	800	x	$\frac{800}{x}$
Airplane	800	$6x$	$\frac{800}{6x}$

To travel 800 km, the train requires 8 h more than the airplane. The equation shown below represents this relationship.

$$\frac{800}{x} - \frac{800}{6x} = 8$$

Numerical Response

The average speed of the train, to the nearest tenth of a kilometre per hour, is _____ km/h.